Southern York County School District Instructional Plan

Course/Subject: Pre-Calculus with Trigonometry
Grade Level: 10, 11, 12

Textbook(s)/Instructional Materials Used:

Pre-Calculus Version 4.0; College Preparatory Mathematics; ISBN-13: 978-1-60328-004-4

Dates: End August-September Unit Plan 1: Algebra and Geometry Review

Stage 1 - Desired Results

PA Core State Assessments/Standards:

- CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents.
- CC.2.2.HS.C.1 Use the concept and notation of functions to interpret and apply them in terms of their context.
- CC.2.2.HS.C.2 Graph and analyze functions and use their properties to make connections between the different representations.
- CC.2.2.HS.C.4 Interpret the effects transformations have on functions and find the inverses of functions.
- CC.2.2.HS.C.6 Interpret functions in terms of the situations they model.
- CC.2.2.HS.C.7 Apply radian measure of angle and unit circle to analyze the trigonometric functions.
- CC.2.2.HS.D.10 Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
- CC.2.3.HS.A.14 Apply geometric concepts to model and solve real world problems.

Understanding(s):

Students will understand

- Real life data does not occur as the perfect graph of an equation. We must find the equation that best fits the data so that we can use it to make inferences about unknown events. (CC.2.2.HS.C.1)
- 2. Transformations of a graph can be symbolized using function notation. (C.C.2.2.HS.C.2)
- 3. There are families of functions and their related graphs. (C.C.2.2.HS.C.2)
- Function notation must be analyzed closely because subtle differences in notation can imply important differences in meaning. (CC.2.2.HS.C.1)
- 5. Some mathematical relationships are classified as functions and some are classified more generally as relationships. (C.C.2.2.HS.C.4)
- 6. Point-slope is an equivalent form of a linear equation. (CC.2.2.HS.D.10)
- 7. The laws of sine and cosine can be used to solve non-right triangles. (CC.2.3.HS.A.14)
- 8. Angles can be measured in degrees or radians. (CC.2.2.HS.C.7)

Essential Question(s):

- How are the tabular, graphical, algebraic and descriptive representations of a situation or data connected? (CC.2.2.HS.C.1)
- How are patterns (represented in a table, graph, by a rule, or in a verbal description) useful to us? (C.C.2.2.HS.C.2)
- There are functions all around us every day. Why is a particular relationship a function? How can an algebraic function model a real world situation? What aspects of the real world situation affect the domain, range and continuity, of the algebraic function? (CC.2.2.HS.C.6)
- Why is it important to apply problem-solving strategies to understanding a problem before actually doing it? (CC.2.3.HS.A.14)
- Why do we learn techniques for simplifying algebraic expressions? (CC.2.1.HS.F.1)

Learning Objectives:

Students will know...

- The unique relationship between a function and its inverse.
- The meaning of function notation for arithmetic operations, compositions, and inverses of functions.
- The basic properties of 6 parent graphs: linear, quadratic, cubic, square root, absolute value, exponential, reciprocal.
- The point-slope form of an equation.
- That radian measure is an alternative way to measure angles.

- Explain their thinking to their classmates.
- Write simple programs for the TI-83
- Interpret radian measure.
- Find a line of best fit.
- Interpret and graph transformations of functions that are written in function notation.
- Interpret and graph transformations of nonparent functions.
- Use the laws of sine and cosine to solve nonright triangles.

Dates: September-October Unit Plan 2: Area Under a Curve

Stage 1 - Desired Results

PA Core State Assessments/Standards:

- CC.2.1.HS.F.5 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
- CC.2.2.HS.C.1 Use the concept and notation of functions to interpret and apply them in terms of their context.
- CC.2.2.HS.C.3 Write functions or sequences that model relationships between two quantities.
- CC.2.2.HS.C.5 Construct and compare linear, quadratic, and exponential models to solve problems.
- CC.2.2.HS.D.2 Write expressions in equivalent forms to solve problems.
- CC.2.2.HS.D.9 Use reasoning to solve equations and justify the solutions method.
- CC.2.3.HS.A.14 Apply geometric concepts to model and solve real world problems.

Understanding(s):

Students will understand...

- Many functions cannot be described with one rule and we need to define them in pieces. (CC.2.2.HS.C.1)
- 2. How to write and interpret summation notation. (CC.2.2.HS.C.3)
- There are several equivalent calculations for finding the area under a curve. (CC.2.3.HS.A.14)
- You can increase your accuracy of calculations of area under a curve by adding up the areas of very small rectangles. (CC.2.2.HS.D.2)

Essential Question(s):

- Is it possible to find the area under an irregular curve on a graph? (CC.2.1.HS.F.5)
- What does the area under a curve mean? (CC.2.2.HS.D.9)
- How do we create models for data in which the rules change for different segments of the domain? (CC.2.2.HS.C.5)

Learning Objectives:

Students will know...

- Area under the curve is an approximation that can be made more accurate by making the rectangles ore trapezoids thinner.
- Some functions are defined by rules for different domain values.
- Piecewise functions can be transformed just like the parent graphs.
- The meaning of periodic functions.

Students will be able to:

- Write a summation notation for a given series.
- Interpret summation notation.
- Approximate the area under a curve on the coordinate plane using a variety of methods.
- Find the area under a curve on the coordinate plane using a series of areas of rectangles.
- Investigate a piecewise function.
- Write simple programs for the TI-83

Dates: October-November Unit Plan 3: Exponentials and Logarithms

Stage 1 - Desired Results

PA Core State Assessments/Standards:

- CC.2.2.HS.C.2 Graph and analyze functions and use their properties to make connections between the different representations.
- CC.2.2.HS.C.4 Interpret the effects transformations have on functions and find the inverses of functions.
- CC.2.2.HS.C.5 Construct and compare linear, quadratic, and exponential models to solve problems.
- CC.2.2.HS.C.6 Interpret functions in terms of the situations they model.
- CC.2.2.HS.D.2 Write expressions in equivalent forms to solve problems.
- CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships.
- CC.2.2.HS.D.8 Apply inverse operations to solve equations or formulas for a given variable.
- CC.2.2.HS.D.9 Use reasoning to solve equations and justify the solutions method.

Understanding(s):

Students will understand...

- 1. Logarithms are the inverse of exponential functions. (CC.2.2.HS.C.2)
- 2. Properties of logarithms are consistent with properties of exponents. (C.C.HS.D.7)
- A multiplier inside a function has a different effect on a graph than a multiplier outside of a function. (CC.2.2.HS.C.4)
- 4. Logarithms enable us to solve exponential equations and exponential expressions enable us to solve logarithmic equations. (CC.2.2.HS.C.6)

Learning Objectives:

Students will know...

- That the output of a logarithmic function is an exponent.
- That the input to a logarithmic function is the result of a base to a power.
- Some transformations are equivalent for certain kinds of functions.
- The meaning of a multiplier inside a function.

Essential Question(s):

- If every operation has an inverse, what is the inverse of raising a value to the xpower? (CC.2.2.HS.D.8)
- What transformation is caused by applying a multiplier inside a function? (CC.2.2.HS.D.9)
- Are some transformations equivalent? Does the answer to that question depend on the original function? (C.C.2.2.HS.D.2)
- What kinds of real life data are modeled by exponential and logarithmic functions? (CC.2.2.HS.C.5)

Students will be able to:

- Graph horizontal stretches and compressions.
- Graph logarithmic functions and describe their properties.
- Solve exponential equations using logarithms.
- Simplify logarithmic expressions using properties of logarithms.
- Apply exponential functions to model real world situations.

Dates: November-December Unit Plan 4: Circular Functions

Stage 1 - Desired Results

PA Core State Assessments/Standards:

- CC.2.2.HS.C.2 Graph and analyze functions and use their properties to make connections between the different representations.
- CC.2.2.HS.C.4 Interpret the effects transformations have on functions and find the inverses of functions.
- CC.2.2.HS.D.2 Write expressions in equivalent forms to solve problems.
- CC.2.2.HS.D.7 Create and graph equations or inequalities to describe numbers or relationships.
- CC.2.2.HS.D.8 Apply inverse operations to solve equations or formulas for a given variable.
- CC.2.2.HS.D.9 Use reasoning to solve equations and justify the solutions method.
- CC.2.2.HS.D.10 Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.

Understanding(s):

Students will understand...

- 1. A multiplier inside a function has a different effect on a graph than a multiplier outside of a function. (CC.2.2.HS.C.4)
- 2. The unit circle is the basis for plotting points on the sinusoidal curve. (CC.2.2.HS.C.7)
- 3. Each trig ratio has an algebraic curve associated with it. (CC.2.2.HS.C.8)
- 4. Proving identities is an algebraic proof that has structure, logic, and organization. (CC.2.2.HS.C.9)
- 5. Repeating data can be modeled with trig functions. (CC.2.2.HS.C.8)

Essential Question(s):

- Why do we use radian measure in advanced algebra courses to measure angles? (CC.2.2.HS.C.7)
- What kind of function can be used to model data that repeats over time? (CC.2.2.HS.C.8)
- What transformation is caused by applying a multiplier inside a function? (CC.2.2.HS.C.2)
- Are some transformations equivalent? Does the answer to that question depend on the original function? (CC.2.2.HS.D.2)
- How do rules and properties of algebra enable us to prove equalities between two expressions? (CC.2.2.HS.D.10)

Learning Objectives:

Students will know...

- The characteristics of trig functions.
- The trig ratios for the 16 special angles on the unit circle.
- The meanings of the reciprocal trig functions.
- The graphical meaning of adding functions

Students will be able to:

- Graph horizontal stretches and compressions.
- Graph any of the 6 trig functions with transformations: af(b(x - h)) + k
- Model given circular or repeating data using trig functions.
- Prove simple trig identities.
- Solve one and two-step trig equations.

Dates: December-January

Unit Plan 5: Introduction to Limits

Stage 1 - Desired Results

PA Core State Assessments/Standards:

- CC.2.2.HS.C.2 Graph and analyze functions and use their properties to make connections between the different representations.
- CC.2.2.HS.C.5 Construct and compare linear, quadratic, and exponential models to solve problems.
- CC.2.2.HS.D.2 Write expressions in equivalent forms to solve problems.
- CC.2.2.HS.D.4 Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs.
- CC.2.2.HS.D.6 Extend the knowledge of rational functions to rewrite in equivalent forms.
- CC.2.2.HS.D.10 Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
- CC.2.4.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable.

Understanding(s):

Students will understand...

- 1. What a limit is as x approaches a value or as it approaches infinity. (CC.2.2.HS.C.2)
- 2. Some basic relationships between limits and vertical and horizontal asymptotes. (CC.2.2.HS.C.5)
- 3. Reciprocal graphs are created by taking the reciprocal of the y values of a given function. (CC.2.2.HS.C.2)
- 4. The relationship between x intercepts of a function and vertical asymptotes of its reciprocal. (CC.2.2.HS.D.4)

Essential Question(s):

- Are some transformations equivalent? Does the answer to that question depend on the original function? (C.C.2.2.HS.D.2)
- How do rules and properties of algebra enable us to prove equalities between two expressions? (C.C.2.2.HS.D.2)
- How do functions behave as x approaches a value or infinity? (CC.2.2.HS.C.2)
- What does the reciprocal of a function look like? ((CC.2.2.HS.D.10)

- Rational functions can be re-written just as improper fractions can be transformed to mixed numbers and that this strategy makes graphing the functions much easier. (CC.2.2.HS.D.6)
- 6. The mathematical definition of continuity is based on the mathematical idea of limits. (CC.2.2.HS.D.10)

 How are the tabular, graphical, algebraic and descriptive representations of a situation or data connected? (CC.2.4.HS.B.1)

Learning Objectives: Students will know...

- The meaning of limits in a variety of contexts.
- That vertical asymptotes present a situation where a limit will not exist for a particular value.
- The meaning of horizontal asymptotes as a limit of the function as x approaches infinity.
- How to define continuity in terms of limits.
- The form of a direct and inverse variation.
- The meanings of the reciprocal trig functions.

Students will be able to:

- Sketch and simplify rational functions.
- Work with direct and inverse variation.
- Graph reciprocal functions, specifically secant and cosecant.
- Find limits at specific points and at infinity.

Dates: January-February

Unit Plan 6: Extending Periodic Functions

Stage 1 - Desired Results

PA Core State Assessments/Standards:

- CC.2.2.HS.C.2 Graph and analyze functions and use their properties to make connections between the different representations.
- CC.2.2.HS.C.4 Interpret the effects transformations have on functions and find the inverses of functions.
- CC.2.2.HS.C.8 Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs.
- CC.2.2.HS.D.2 Write expressions in equivalent forms to solve problems.
- CC.2.2.HS.D.10 Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
- CC.2.3.HS.A.3 Verify and apply theorems involving similarity as they relate to geometric figures.
- CC.2.3.HS.A.14 Apply geometric concepts to model and solve real world problems.
- CC.2.4.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable.

Understanding(s):

Students will understand...

- Inverse Sine and Cosine functions have restricted ranges which means that solving trig equations requires analysis beyond the solution provided by a calculator. (CC.2.2.HS.C.4)
- 2. Why SSA is not a triangle congruence theorem. (CC.2.3.HS.A.3)
- 3. There are trig identities for angle sums and differences. (CC.2.3.HS.A.14)
- 4. Repeating data can be modeled with trig functions. (CC.2.2.HS.C.8)
- 5. A multiplier inside a function has a different effect on a graph than a multiplier outside of a function. (CC.2.2.HS.C.4)
- 6. Each trig ratio has an algebraic curve associated with it. (CC.2.2.HS.C.8)

Essential Question(s):

- Are some transformations equivalent? Does the answer to that question depend on the original function? (C.C.2.2.HS.D.2)
- How do rules and properties of algebra enable us to prove equalities between two expressions? (C.C.2.2.HS.D.2)
- How do functions behave as x approaches a value or infinity? (CC.2.2.HS.C.2)
- What does the reciprocal of a function look like? (CC.2.2.HS.D.10)
- How are the tabular, graphical, algebraic and descriptive representations of a situation or data connected? (CC.2.4.HS.B.1)

Learning Objectives: Students will know...

- The graph and restricted ranges of inverse sine and cosine.
- Analysis is necessary whenever using the inverse sine and cosine functions to solve equations.
- The angle sum and difference formulas
- The form of a trig equation when there is a horizontal shift and stretch.
- The characteristics of trig functions.

Students will be able to:

- Graph any of the 6 trig functions with transformations: af(b(x - h)) + k
- Find multiple solutions to trig equations using the calculator, symmetry of graphs, and analysis of the period of the function.
- Model given circular or repeating data using trig functions.
- Prove a variety of trig identities.
- Solve one and two-step trig equations.

Dates: February-March Unit Plan 7: Algebra for College

Stage 1 - Desired Results

PA Core State Assessments/Standards:

- CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents.
- CC.2.1.HS.F.4 Use units as a way to understand problems and to guide the solution of multi-step problems
- CC.2.2.HS.C.2 Graph and analyze functions and use their properties to make connections between the different representations.
- CC.2.2.HS.C.3 Write functions or sequences that model relationships between two quantities.
- CC.2.2.HS.C.6 Interpret functions in terms of the situations they model.
- CC.2.2.HS.D.2 Write expressions in equivalent forms to solve problems.
- CC.2.2.HS.D.9 Use reasoning to solve equations and justify the solutions method.
- CC.2.3.HS.A.14 Apply geometric concepts to model and solve real world problems.
- CC.2.4.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable.

Understanding(s):

Students will understand...

- We learn to simplify what appears to be a complex problem so that it is made more manageable. Specifically, students will use u-substitution when solving systems of equations. (CC.2.2.HS.D.9)
- Pascal's triangle makes expanding binomials much easier and far less time intensive. (CC.2.3.HS.A.14)
- Word problems, such as related rate problems, should be analyzed with a methodical set of problem-solving steps before any attempt is made to solve the problem. (CC.2.1.HS.F.4)
- 4. Graphs can be categorized by their concavity and symmetry. (CC.2.2.HS.C.2)
- 5. A series is a sum of a sequence. (CC.2.2.HS.C.3)

Essential Question(s):

- How do rules and properties of algebra enable us to prove equalities between two expressions? (C.C.2.2.HS.D.2)
- How are the tabular, graphical, algebraic and descriptive representations of a situation or data connected? (CC.2.4.HS.B.1)
- There are functions all around us every day. Why is a particular relationship a function? How can an algebraic function model a real world situation? What aspects of the real world situation affect the domain, range and continuity, of the algebraic function? (CC.2.2.HS.C.6)
- Why is it important to apply problem-solving strategies to understanding a problem before actually doing it? (CC.2.2.HS.C.6)
- Why do we learn techniques for simplifying algebraic expressions? (CC.2.1.HS.F.1)

Learning Objectives: Students will know...

Pascal's triangle.

 The important steps in analyzing a challenging word problem.

- Expand binomials using Pascal's triangle.
- Use u-substitution to solve complicated systems of equations.

 The definition of concavity and odd/even/neither symmetry.

- Analyze graphs and identify symmetry and concavity intervals.
- Calculate a finite series.
- Solve related rate problems graphically.

Dates: March-April Unit Plan 8: More on Limits

Stage 1 – Desired Results

PA Core State Assessments/Standards:

- CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents.
- CC.2.1.HS.F.2 Apply properties of rational and irrational numbers to solve real world or mathematical problems.
- CC.2.1.HS.F.4 Use units as a way to understand problems and to guide the solution of multi-step problems.
- CC.2.2.HS.C.3 Write functions or sequences that model relationships between two quantities.
- CC.2.2.HS.C.6 Interpret functions in terms of the situations they model.
- CC.2.2.HS.D.2 Write expressions in equivalent forms to solve problems.
- CC.2.2.HS.D.9 Use reasoning to solve equations and justify the solutions method.
- CC.2.2.HS.D.10 Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
- CC.2.4.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable.

Understanding(s):

Students will understand...

- Word problems, such as related rate problems, should be analyzed with a methodical set of problem-solving steps before any attempt is made to solve the problem. (CC.2.1.HS.F.4)
- 2. Algebraic manipulation is often necessary to evaluate a limit algebraically. (CC.2.2.HS.D.2)
- Undefined domain values are not always represented graphically as v. asymptotes. Sometimes they are holes of discontinuity in the graph. (CC.2.2.HS.D.10)
- 4. Limits can be used to find the sum of series. (CC.2.2.HS.C.3)
- 5. The irrational numbers pi and e are applications of limits. (CC.2.1.HS.F.2)

Essential Question(s):

- How do rules and properties of algebra enable us to prove equalities between two expressions? (C.C.2.2.HS.D.2)
- How are the tabular, graphical, algebraic and descriptive representations of a situation or data connected? (CC.2.4.HS.B.1)
- There are functions all around us every day. Why is a particular relationship a function? How can an algebraic function model a real world situation? What aspects of the real world situation affect the domain, range and continuity, of the algebraic function? (CC.2.2.HS.C.6)
- Why is it important to apply problem-solving strategies to understanding a problem before actually doing it? (CC.2.2.HS.C.6)
- Why do we learn techniques for simplifying algebraic expressions? (CC.2.1.HS.F.1)
- How do functions behave as x approaches a value or infinity? (CC.2.2.HS.D.9)

Learning Objectives: Students will know...

- The definitions of geometric and harmonic series and the Fibonacci sequence.
- The meaning of e and its applications.
- That undefined domain values can appear as a v. asymptote or a hole on a graph.
- How to identify a dominant term.
- The important steps in analyzing a challenging word problem.

- Use dominant terms to find limits at infinity.
- Find limits of rational functions.
- Apply the squeeze theorem to approximate pi.
- Apply the natural base, e, to solve exponential applications.
- Find sums of infinite geometric series.

| Investigate the Harmonic Series and the Fibonacci Sequence. |
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Dates: April-May Unit Plan 9: Rates of Change

Stage 1 - Desired Results

PA Core State Assessments/Standards:

- CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents.
- CC.2.1.HS.F.4 Use units as a way to understand problems and to guide the solution of multi-step problems.
- CC.2.2.HS.C.6 Interpret functions in terms of the situations they model.
- CC.2.2.HS.D.1 Interpret the structure of expressions to represent a quantity in terms of its context.
- CC.2.2.HS.D.2 Write expressions in equivalent forms to solve problems.
- CC.2.2.HS.D.9 Use reasoning to solve equations and justify the solutions method.
- CC.2.2.HS.D.10 Represent, solve, and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
- CC.2.4.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable.

Understanding(s):

Students will understand...

- The average rate of change becomes an instantaneous rate of change between two points as the limit of the difference between the two x-coordinates approaches 0. (CC.2.2.HS.D.9)
- 2. Using a distance equation, we can find velocity at a given moment using derivatives and using a velocity equation, we can find distance using integrals. (These are the two fundamental ideas of calculus.) (CC.2.2.HS.D.1)
- Word problems, such as related rate problems, should be analyzed with a methodical set of problem-solving steps before any attempt is made to solve the problem. (CC.2.1.HS.F.4)
- 4. Algebraic manipulation is often necessary to evaluate a limit algebraically. (CC.2.2.HS.D.2)
- Undefined domain values are not always represented graphically as v. asymptotes. Sometimes they are holes of discontinuity in the graph. (CC.2.2.HS.D.10)

Essential Question(s):

- How do rules and properties of algebra enable us to prove equalities between two expressions? (C.C.2.2.HS.D.2)
- How are the tabular, graphical, algebraic and descriptive representations of a situation or data connected? (CC.2.4.HS.B.1)
- There are functions all around us every day. Why is a particular relationship a function? How can an algebraic function model a real world situation? What aspects of the real world situation affect the domain, range and continuity, of the algebraic function? (CC.2.2.HS.C.6)
- Why is it important to apply problem-solving strategies to understanding a problem before actually doing it? (CC.2.2.HS.C.6)
- Why do we learn techniques for simplifying algebraic expressions? (CC.2.1.HS.F.1)
- How do functions behave as x approaches a value or infinity? (CC.2.2.HS.D.9)
- How are functions behaving at specific points? (CC.2.2.HS.D.1)

Learning Objectives:

Students will know...

- The important steps in analyzing a challenging word problem.
- The difference between AROC and IROC.
- The definition of derivative.
- IROC is an application of limits.
- The relationship between velocity and distance functions.

- Calculate rates of change for a variety of different situations.
- Estimate instantaneous rates of change.
- Use limits to find IROC.